An Approximate Dispersion Formula of Microstrip Lines for Computer-Aided Design of Microwave Integrated Circuits

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***Abstract*—A simple approximate formula for the computer-aided design of the dispersion property of microstrip lines is reported in this paper which well fits the calculated curves based on a rigorous analysis.**

I. Introduction

Microstrip lines have been widely used as basic components of microwave integrated circuits. The TEM approximation [1] is employed, in many cases, for analyzing the propagation modes of these lines. Two features of the TEM mode are that the wave velocity is independent of frequencies and the longitudinal components of electromagnetic fields are neglected.

When the wavelength becomes comparable to cross-sectional dimensions of the microstrip line, the wave velocity is no longer independent of frequencies. This phenomenon is known as the dispersion property and should be considered in practice. An exact analysis of this property is possible by using Maxwell's equations. The numerical results of an analysis based on the integral equation method [2] agree well with those based on the mode matching method [3] in a wide frequency range. This is an indication of accuracy in both methods.

On the other hand, a simple approximate formula to express the dispersion property is needed for the purpose of desk calculations or the computer aided design of microwave integrated circuits. Most of rigorous calculation methods including the above two are not appropriate in such applications because they usually require a complicated computer programming. Though a few approximate formulas have been reported based on some physical considerations and experimental data [4]–[7], these empirical formulas have had limited ranges of applicability and inadequate theoretical foundation for confidence in application.

Our approximate dispersion formula to be derived here has a simple form and is based on a rigorous theory in contrast to other approximate formulas.

II. Computation of Propagation Constant

Our computation of the propagation constant of microstrip lines is based on the integral equation method which has been reported in [2]. Fig. 1 shows the transmission line structure treated with this method. The microstrip line is only a special case of Fig. 1. For instance, a good approximation of such structures may be obtained by setting parameters as $h_1=0$, $h_2=h$, $h_3=1000h$, $a=W+8h$, $\epsilon_2=\epsilon^*\epsilon_0$, $\epsilon_3=\epsilon_0$. Fig. 2 shows the evaluated propagation constant of the dominant mode, β, by setting ϵ^* as a parameter, where β_0 is the propagation constant in vacuum. The dispersion or the dependence of β/β_0 on frequencies is observed quite well here. Fig. 3 shows the same quantity by setting W/h as a parameter.

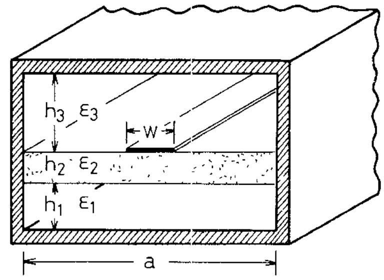

Fig. 1. Transmission lines treated with the integral equation method.

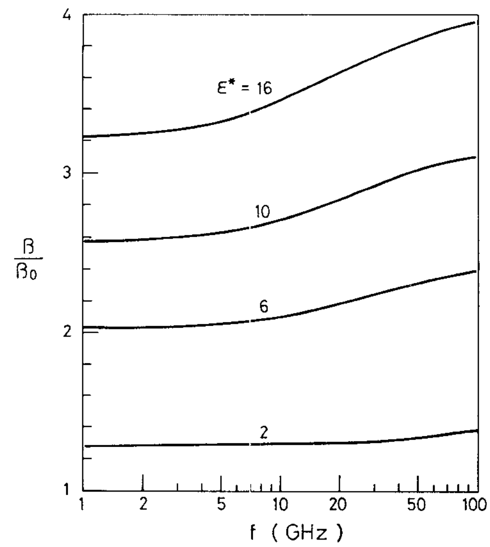

Fig. 2. Computed propagation constants for $h=1$ mm, $W/h=1$.

III. Derivation of Approximate Dispersion Formula

Let us find a simple formula to fit these curves with least errors in a wide range of frequencies, dielectric constant, and physical dimensions. The propagation con-

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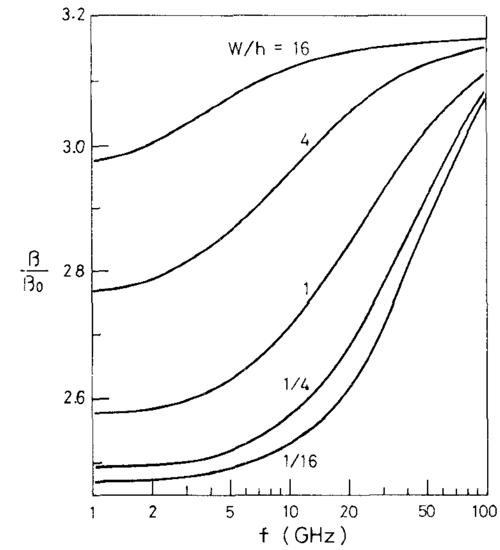

Fig. 3. Computed propagation constants for $h = 1$ mm, $\epsilon^* = 10$.

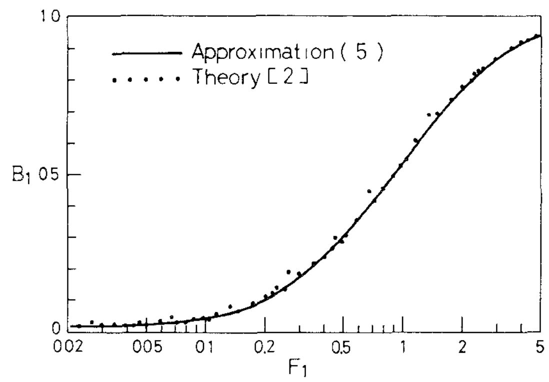

Fig. 4. An approximate logistic curve to fit the theoretical values of propagation constants in Fig. 2.

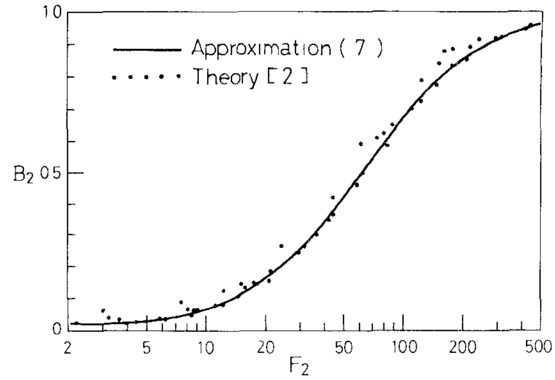

Fig. 5. An approximate logistic curve to fit the theoretical values of propagation constants in Fig. 3.

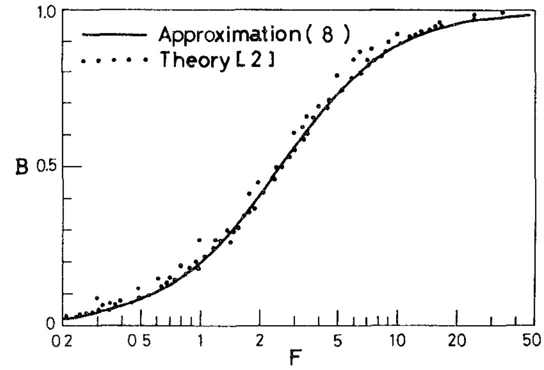

Fig. 6. An approximate logistic curve to fit the theoretical values of propagation constants in Figs. 2 and 3.

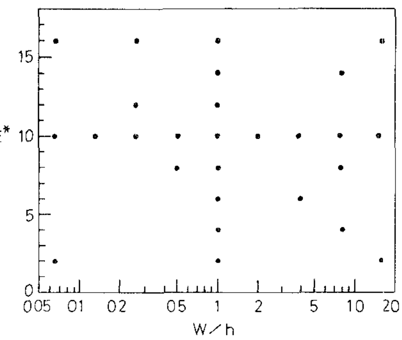

Fig. 7. Sample parameters used to estimate errors of the proposed approximate formula (10).

stant of the dominant mode has the following two extremes: $\beta \rightarrow \beta_{TEM}(f \rightarrow 0)$ and $\beta \rightarrow \beta_0\sqrt{\epsilon^*}$ $(f \rightarrow \infty)$. β_{TEM} is the propagation constant obtained with the TEM wave approximation. At first, we define the normalized propagation constant B as

$$B = \frac{\beta/\beta_0 - \beta_{TEM}/\beta_0}{\sqrt{\epsilon^*} - \beta_{TEM}/\beta_0} \tag{1}$$

so as to confine all curves between the two extremes, $B \rightarrow 0 (f \rightarrow 0)$ and $B \rightarrow 1 (f \rightarrow \infty)$. Frequencies are also normalized as

$$F_1 = \frac{f}{f_{TE1}} \tag{2}$$

to collect all curves as only one curve. f_{TE1} is the cutoff frequency of the surface-wave TE_1 mode given by

$$f_{TE1} = \frac{v_0}{4h\sqrt{\epsilon^* - 1}} \tag{3}$$

where v_0 is the velocity of light in vacuum. Fig. 4 shows the result of the normalization drawn on a log scale.

We found a resemblance between the logistic curve given by

$$y(x) = \frac{1}{1+e^{-bx}}, \qquad -\infty < x < \infty \tag{4}$$

and the one in Fig. 4. Considering the logarithmic scale in Fig. 4, we rewrite (4) in a form

$$B_1 = \frac{1}{1 + a_1 F_1^{-b_1}}. \tag{5}$$

Selected values for the parameters, a_1 and b_1, to fit the curves in Fig. 4 are $a_1 = 0.785$ and $b_1 = 1.5$.

We also normalized frequencies in Fig. 3 as

$$F_2 = f\left[0.5 + \left\{1 + 2\log\left(1 + \frac{W}{h}\right)\right\}^2\right] \tag{6}$$

so that all curves shrink to almost one curve. The result of the normalization is shown in Fig. 5. Again, this curve can be expressed by a logistic curve

$$B_2 = \frac{1}{1 + a_2 F_2^{-b_2}}. \tag{7}$$

Selected parameters to fit the curve in Fig. 5 are $a_2 = 0.785 \times 10^{-1.5}$ and $b_2 = 1.5$.

The above two curves, (5) and (7), are combined together and expressed by

$$B = (1/1 + aF^{-b}) \tag{8}$$

where the normalized frequency is given by

$$F = (F_1 F_2 / f) \tag{9}$$

and selected values of parameters are $a = 4$ and $b = 1.5$.

As a conclusion, we obtained an approximate dispersion formula:

$$\frac{\beta}{\beta_0} = \frac{\sqrt{\epsilon^*} - \dfrac{\beta_{TEM}}{\beta_0}}{1 + 4F^{-1.5}} + \frac{\beta_{TEM}}{\beta_0} \tag{10}$$

where

$$F = \frac{4h\sqrt{\epsilon^* - 1}}{\lambda_0}[0.5 + \{1 + 2\log(1 + \frac{W}{h})\}^2]. \tag{11}$$

The estimated values of the propagation constants by the approximate formula (10) and the theoretical values given in Figs. 2 and 3 are compared as a solid line and dots in Fig. 6. The differences between these values were found to be within 1 percent for sampled parameters shown as dots in Fig. 7 in a wide frequency range. The above formula assumes the knowledge of β_{TEM} but a simple formula of β_{TEM} can be easily found in literatures such as Wheeler [8].

References

[1] E. Yamashita *et al.*, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, p. 251, Apr. 1968.

[2] E. Yamashita *et al.*, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, p. 195, Apr. 1976.

[3] G. Kowalski *et al.*, *Arch. Elek. Übertragung*, vol. 107, p. 163, Apr. 1971.

[4] O. P. Jain *et al.*, *Electron. Lett.*, vol. 7, p. 405, July 1971.

[5] W. J. Getsinger, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, p. 34, Jan. 1973.

[6] M. V. Schneider, *Proc. IEEE*, vol. 60, p. 144, Jan. 1972.

[7] H. J. Carlin, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, p. 589, Sept. 1973.

[8] H. A. Wheeler, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, p. 172, Mar. 1965.

3-GHz 15-W Silicon Bipolar Transistors

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***Abstract*—Silicon bipolar transistors delivering 15-W CW output power with 4.8-dB gain and 38-percent collector efficiency have been developed at 3 GHz. The transistors have been fabricated by boron ion implantation for base region and arsenic diffusion from doped polysilicon for emitter region. Chemical dry-etching techniques for fine patterning and internal-matching techniques have been applied.**

I. Introduction

IN SPITE OF recent progress in power GaAs FET's, silicon bipolar transistors can still be considered the best candidate for solid-state power devices in view of their higher output power and cost performance [1]. Recently the output power capability of silicon bipolar transistors has been extended up to X band and a CW output power of 1.5 W at 10 GHz is reported [2]. However, most efforts are concentrated upon frequencies below 5 GHz and CW output powers of 60 W at 2 GHz and 6 W at 5 GHz are obtained [3]. Although bipolar transistors with 8-W CW output power are commercially available at 3 GHz, further efforts towards increasing CW output power are, to the authors' knowledge, not reported.

On the other hand, the maximum attainable output power is limited not only by the device capability, but also by the device–circuit interface. Hence internal matching procedures are generally used for the improvement of device–circuit interface [4].

This paper reports on recently developed internally matched 3-GHz silicon bipolar power transistors that deliver 15-W CW output power with 4.8-dB gain under class

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